Σ^- Lifetime
Teacher Notes

This activity gives students an opportunity to measure the mean lifetime of a particle that lasts long enough to travel a measurable distance in a bubble chamber. It shows the probabilistic nature of particle lifetimes and a typical lifetime of a weakly decaying particle. Particles that decay through the electromagnetic or strong interactions don't travel a measurable distance before decaying. Their lifetimes must be calculated with the Heisenberg uncertainty relation $\Delta E \Delta t \geq h/4\pi$ and their measured energy spread ΔE.

Student Learning Objectives

1. *Lifetime of moving particles is related to track length in a bubble chamber.*
2. *Particle decay is a random process, and mean particle lifetime can be measured.*
3. *Greater numbers of measurements of a random process yield greater precision of the mean value of the process.*

Essential Knowledge

1. *Familiarity with relations between relativistic energy, mass, momentum, kinetic energy, and speed of a particle*
2. *Acquaintance with exponential function and natural logarithms*
3. *Familiarity with curve fitting on a calculator or computer spread sheet*

The Teacher's Solution below contains typical measured valued and calculations printed in **bold-faced** type.

(Meets LO 1 with values provided in Table 2 as it relates to track length-lifetime relations for a particle)
(Meets LO 2 as it relate to measurement of particle track length)
(Meets LO 3 as it relate to calculation of uncertainty related to number of measurements)

TEACHER'S SOLUTION

Most of the particles discovered during the 20th century are unstable and spontaneously decay with very short mean lifetimes into less massive particles. Our goal here is to determine the lifetime of one such particle, the Σ^- particle. To produce the Σ^- particles whose tracks are shown in Figures 1 and 2, high-energy protons from the CERN Proton Synchrotron crashed into a stationary target. Among the particles produced were negative kaons (K^-). These kaons were then selected and directed to the Saclay 80-cm bubble chamber. Many of the kaons incident on the chamber had low enough energy that they stopped before interacting with protons in the chamber. In the photos in Figures 1 and 2 a negative kaon (K^-) entered from the bottom and left a trail of ions around which hydrogen bubbles formed. The K^- came to rest and interacted with a proton (also essentially at rest) from the liquid hydrogen. The photos show production of a Σ^- and a π^+, one of several possible results of the kaon-proton interaction.

Explain why you expect the Σ^- and π^+ to have equal and opposite momenta.

The kaon and proton is at rest before the interaction had zero total momentum. Total momentum after interaction must be zero (momentum conservation). Thus, the two particles resulting from the interaction must have equal and opposite momenta: $\boldsymbol{p}_K + \boldsymbol{p}_p = 0 = \boldsymbol{p}_\Sigma + \boldsymbol{p}_\pi \Rightarrow$

$\boldsymbol{p}_\Sigma = -\boldsymbol{p}_\pi$

Use energy conservation to **show that** Σ^- and π^+ momentum must each be 173 MeV.

Energy conservation: $m_K + m_p = E_\Sigma + E_\pi$

$$493.7\text{ MeV} + 938.3\text{ MeV} = \sqrt{(1197MeV)^2 + (p)^2} + \sqrt{(140MeV)^2 + (p)^2}$$

$$1432\text{ MeV} - \sqrt{(1197MeV)^2 + p^2} = \sqrt{(140MeV)^2 + p^2}$$

Square both sides of the equation and solve for *p*.

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$p = p_\Sigma = p_\pi = 173$ **MeV**

Calculate the **energy, kinetic energy,** and **speed** of the Σ^- when it is produced.

$E_\Sigma^2 = m_\Sigma^2 + p_\Sigma^2 = (1197\text{ MeV})^2 + (173\text{ MeV})^2 \Rightarrow E_\Sigma = 1209\text{ MeV}$

$KE_\Sigma = E_\Sigma - m_\Sigma = (1209\text{ MeV} - 1197\text{ MeV}) \Rightarrow KE_\Sigma = 12\text{ MeV}$

$v_\Sigma = p_\Sigma/E_\Sigma = (173\text{ MeV})/(1209\text{ MeV}) \Rightarrow v_\Sigma = 0.143$

Calculate the **energy, kinetic energy,** and **speed** of the π^+ when it is produced.

$E_\pi^2 = m_\pi^2 + p_\pi^2 = (140\text{ MeV})^2 + (173\text{ MeV})^2 \Rightarrow E_\pi = 223\text{ MeV}$

$$KE_\pi = E_\pi - m_\pi = (223\text{ MeV} - 140\text{ MeV}) \quad \Rightarrow \quad KE_\pi = 83\text{ MeV}$$

$$v_\pi = p_\pi/E_\pi = (173\text{ MeV})/(223\text{ MeV}) \quad \Rightarrow \quad v_p = 0.776$$

The π^+ and the Σ^- are unstable particles. About 99.99% of all positive pions decay into a positive muon and a muon neutrino ($\pi^+ \rightarrow \mu^+ + \nu_\mu$). The pion mean lifetime is $\tau_\pi = 2.60 \times 10^{-8}$ s. About 99.85% of all Σ^- particles decay into a neutron and a negative pion ($\Sigma^- \rightarrow n^0 + \pi^-$). In this exercise you will measure the Σ^- mean lifetime (τ_{Σ^-}).

Particle decay seems to be a random process like flipping coins or rolling dice. Imagine a large collection of *N* coins. All coins are flipped at once, and after the flip each coin that shows heads is removed from the collection. The remaining coins are flipped again, and the process of removing coins that show heads is repeated. As this process continues, the number of coins remaining decreases, and the number that show heads after each flip decreases roughly in the same proportion. For a single coin, the outcome of any one flip (heads or tails) is not easily predicted. However, for a large number of flips of a fair coin or one flip of a large number of fair coins nearly half of the coins flipped will produce heads and nearly half will produce tails. When starting with a large number of coins (N_0), we expect that half the coins after any flip will show heads and be removed. We can represent the number of coins (N) remaining after a number of flips (n) by

$$N = N_0\left(\frac{1}{2}\right)^n = N_0(2^{-1})^n = N_0(e^{-\ln 2})^n = N_0 e^{-0.693n} = N_0 e^{-\lambda n}, \text{ where } \lambda = 0.693 .$$

Suppose you have a large collection of N_0 fair 6-sided dice. After each shake of the dice, those showing one spot on the top face are removed.
Write an expression for the number (N) of dice remaining after n shakes.
Calculate the value of the decay constant λ.

$$N = N_0\left(\frac{5}{6}\right)^n = N_0(1.2^{-1})^n = N_0(e^{-\ln 1.2})^n = N_0 e^{-0.182n} = N_0 e^{-\lambda n}, \textbf{ where } \boldsymbol{\lambda} \textbf{ = 0.182}$$

We assume a random process of the type indicated above represents particle decay. In this model for unstable particles, a constant fraction of the particles present at any time ($\Delta N/N$) will decay and be removed from the collection during some specified time interval Δt, *i.e.,* **($\Delta N/N$)/Δt = λ = decay constant**. The value of λ depends on the nature of the particle. For a system starting with N_0 particles at time $t = 0$, the number of particles remaining at any later time is given by the expression

$$N = N_0 e^{-\lambda t} = N_0 e^{-t/\tau}, \text{ where } 1/\lambda = \tau = \text{mean lifetime.}$$

By measuring $(\Delta N/N)/\Delta t = \lambda$ several times for a random collection of one type of particle, one can determine an average value for λ and, consequently, an average value for τ, the mean lifetime of the particle. That is the goal of this exercise.

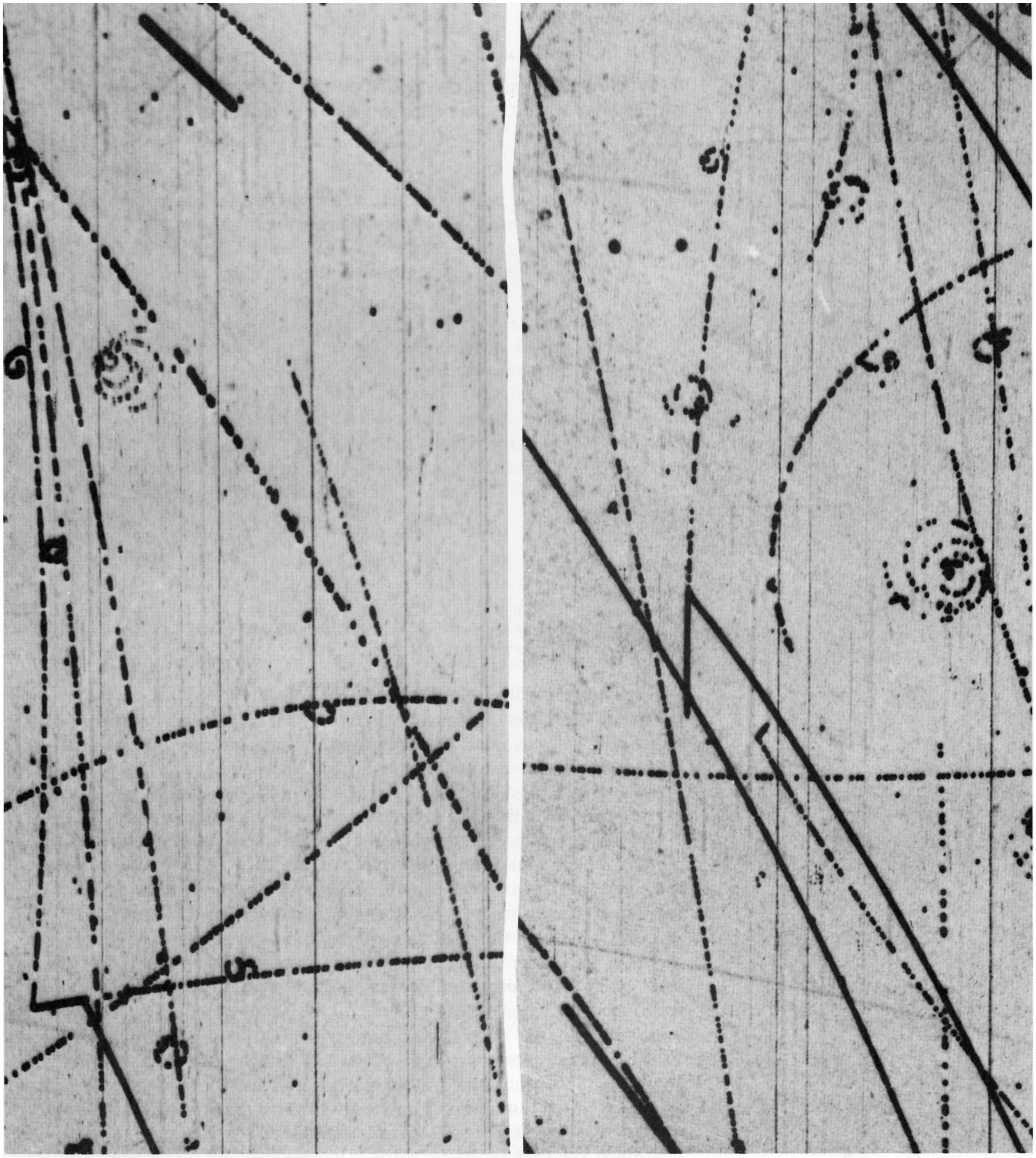


Figure 1: Photo (tracks twice actual linear size) from film of a Stopping K$^-$ run at CERN in the Saclay 80-cm Hydrogen Bubble Chamber. Adapted from H. Whiteside, *Elementary Particles*, 1971, photo from of the University of Maryland High Energy Physics Group.

Notes: Photo on left is analyzed in the Inelastic Scattering exercise.
The S$^-$ in the photo on the right slows to a stop, interacts with a proton and produces neutral particles that leave no tracks.

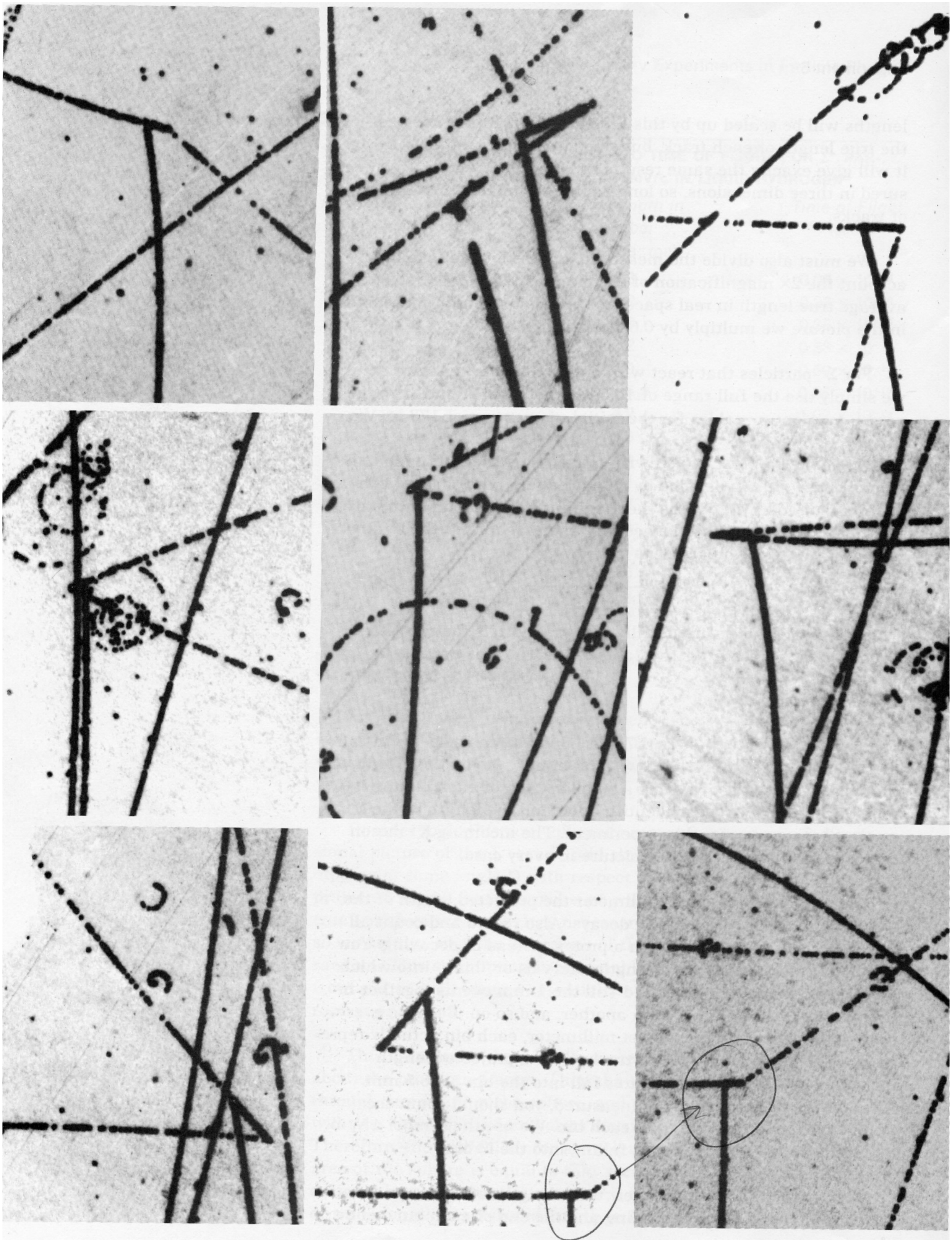


Figure 2: Segments of nine photos (tracks twice actual linear size) from film of a Stopping K^- run at CERN in the Saclay 80-cm Hydrogen Bubble Chamber. (From H. Whiteside, *Elementary Particles*, 1971.) Circled S^- interaction in bottom center is displayed in bottom right photo.

The **photos in Figures 1 and 2 present a small random selection from a large collection of images of the interaction $K^- + p^+ \rightarrow \Sigma^- + \pi^+$ in the Saclay 80-cm liquid hydrogen bubble chamber at twice its actual size**; *i.e.*, 2 cm on photo represents 1 cm on bubble chamber surface. A nearly uniform magnetic field was directed toward the viewer perpendicular to the plane of the photo. The strong magnetic field (B = 1.7 tesla) produced curvature in the tracks of charged particles moving in the photo plane. In most cases, the Σ^- decays in flight by the process $\Sigma^- \rightarrow n^0 + \pi^-$. The Σ^- will occasionally last long enough to lose all its kinetic energy by ionizing hydrogen atoms along its path in the chamber. (The path length will be about one centimeter in the chamber (2 cm in the photos) for Σ^- particles with initial momentum p = 173 MeV.) When that happens, the Σ^- can interact with a stationary proton to form neutral particles that leave no tracks.

Identify the Σ^- tracks in Figures 1 and 2 and **measure** their lengths (L_M) to the nearest mm. Record the values in the table below.

Table 1. L_M Measured Σ^- Track Lengths (mm)

Fig. 1 Left	Fig. 1 Right	Fig. 2 Upper Left	Fig. 2 Upper Center	Fig. 2 Upper Right
9	**22**	**4**	**16**	**8**

Fig. 2 Center Left	Fig. 2 Center Center	Fig. 2 Center Right	Fig. 2 Lower Left	Fig. 2 Lower Center	Fig. 2 Lower Right
4	**2**	**7**	**10**	**14**	**3**

The process by which a moving charged particle slows down while ionizing hydrogen atoms in a liquid hydrogen bubble chamber is well studied. As a particle loses energy (and momentum), its speed decreases. It spends more time in the vicinity of each H atom along its path and produces more H ions. Thus, ion density and energy loss rate and time per mm of track length increase as the particle moves along. For S^- particles with initial momentum of 173 MeV, this relation is embodied in the values for times of flight corresponding to various track lengths listed in Table 1.

The track lengths that you measure in the photos are not likely to be the true track lengths in the bubble chamber since the true tracks are not likely to be in the plane of the photos. The S^- tracks are more likely to be formed at some dip angle (θ) to the photo plane. Then the true track length (L_T) is related to the measured track length (L_M) by the expressions

$$L_M = L_T\cos\theta \qquad \text{and} \qquad L_T = L_M/\cos\theta \,.$$

Since we don't have values for the dip angle for each S^- track, we will adjust each L_M value by dividing it by 2 to compensate for the magnified scale of the photos and dividing by the average value of $\cos\theta$. This should be a reasonable correction to convert L_M values to L_T values. In three dimensions, the average value of $\cos\theta$ is $\pi/4$.

Thus, $L_T = (L_M/2)/(\pi/4) = (2/\pi)L_M = 0.637L_M$.

Table 2. Time of Flight Related to Track Length for S^- Particles with Initial Momentum of 173 MeV

L_M Bin (mm)	L_T Avg. (mm)	Bin Start Time (10^{-10} s)	Bin End Time (10^{-10} s)	Bin Δt (10^{-10} s)	N_i S^- at Bin Start	ΔN_i S^- Decays in Bin
0.5 – 1.5	0.3 – 1.0	0.08	0.23	0.15	**11**	**0**
1.5 – 2.5	1.0 – 1.6	0.23	0.38	0.15	**11**	**1**
2.5 – 3.5	1.6 – 2.2	0.38	0.53	0.15	**10**	**1**
3.5 – 4.5	2.2 – 2.9	0.53	0.69	0.16	**9**	**2**
4.5 – 5.5	2.9 – 3.5	0.69	0.85	0.16	**7**	**0**
5.5 – 6.5	3.5 – 4.2	0.85	1.02	0.17	**7**	**0**
6.5 – 7.5	4.2 – 4.8	1.02	1.19	0.17	**7**	**1**
7.5 – 8.5	4.8 – 5.4	1.19	1.36	0.17	**6**	**1**
8.5 – 9.5	5.4 – 6.1	1.36	1.54	0.18	**5**	**1**
9.5 – 10.5	6.1 – 6.7	1.54	1.73	0.19	**4**	**1**
10.5 – 11.5	6.7 – 7.4	1.73	1.94	0.21	**3**	**0**
11.5 – 12.5	7.4 – 8.0	1.94	2.16	0.22	**3**	**0**
12.5 – 13.5	8.0 – 8.6	2.16	2.39	0.23	**3**	**0**
13.5 – 14.5	8.6 – 9.3	2.39	2.65	0.26	**3**	**1**
14.5 – 15.5	9.3 – 9.9	2.65	2.95	0.30	**2**	**0**

From your record of measured track lengths (L_M) in Table 1, **enter** the number of decays in each bin (ΔN_i) in Table 2. Then **calculate** and **enter** the N_i values for each bin.

Next, **calculate** from Table 2 and **enter** in Table 3 below the decay constant λ_i for each bin (i) from L_M = 2 mm to L_M = 15 mm $\quad \lambda_i = (\Delta N_i/N_i)/\Delta t_i$.

Track lengths less than 2 mm are omitted from the calculation because such short tracks are difficult to identify with reliability. For tracks longer than 15 mm, the S^- particle is likely to have come to rest and interact with a proton before decaying so those track lengths are not representative of decay times. Thus, tracks longer than 15 mm are also omitted from the calculation. Some of the λ_i values will be zero.

Calculate the average λ_i value. Include the zero λ_i values in your average.

Average $\lambda = \lambda_{av}$ = **0.59×10^{10} s^{-1}**

Calculate the mean lifetime (τ) for the S^- particle and the approximate precision of the measurement $\tau/\sqrt{n}$, where n = number of decays included in the calculation of τ.

Mean lifetime $\tau = 1/\lambda_{av}$ = **1.71×10^{-10} s** $\quad \pm \; \tau/\sqrt{n}$ = **0.54×10^{-10} s**

Table 3. Decay Constants for Each Bin

L_M Bin (mm)	$\lambda_i = (\Delta N_i/N_i)/\Delta t_i$ (10^{10} s^{-1})
1.5 – 2.5	**0.61**
2.5 – 3.5	**0.67**
3.5 – 4.5	**1.39**
4.5 – 5.5	**0**
5.5 – 6.5	**0**
6.5 – 7.5	**0.84**
7.5 – 8.5	**0.98**
8.5 – 9.5	**1.11**
9.5 – 10.5	**1.32**
10.5 – 11.5	**0**
11.5 – 12.5	**0**
12.5 – 13.5	**0**
13.5 – 14.5	**1.28**
14.5 – 15.5	**0**

Finally, **plot** N_i vs. t for L_M values of 2 mm to 15 mm. **Fit** an exponential curve to the data, and **compare** λ and τ values from the fit to your calculated λ_{av} and τ values and to standard λ and τ values.

Calculated: λ_{av} = **0.59 x 10^{10} s^{-1}** τ = **(1.71 ± 0.54) x 10^{-10} s**

NOTE:

$$N = N_0 e^{-\lambda t} \quad \Rightarrow \quad \frac{dN}{dt} = -\lambda N_0 e^{-\lambda t} = -\lambda N$$

$$\frac{dN}{Ndt} = -\lambda \quad \Rightarrow \quad \frac{dN/N}{dt} = -\lambda$$

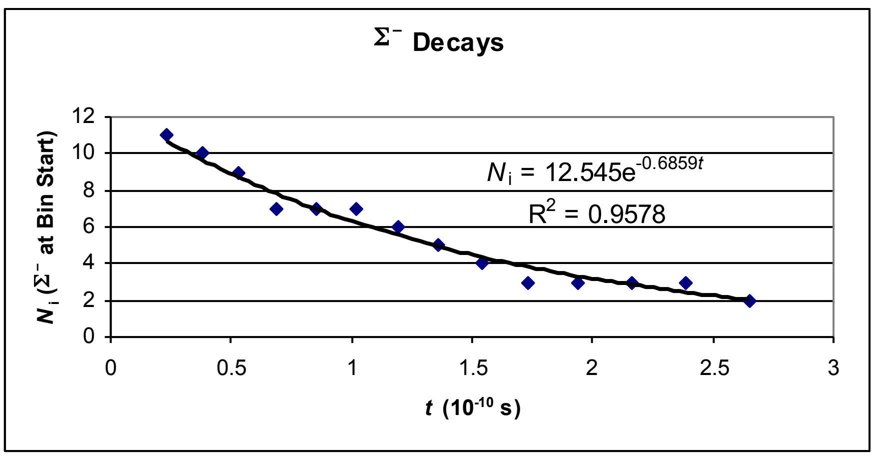
Fit (for N_{i0} = 11): λ = **0.61 x 10^{10} s^{-1}** τ = **1.64 x 10^{-10} s**

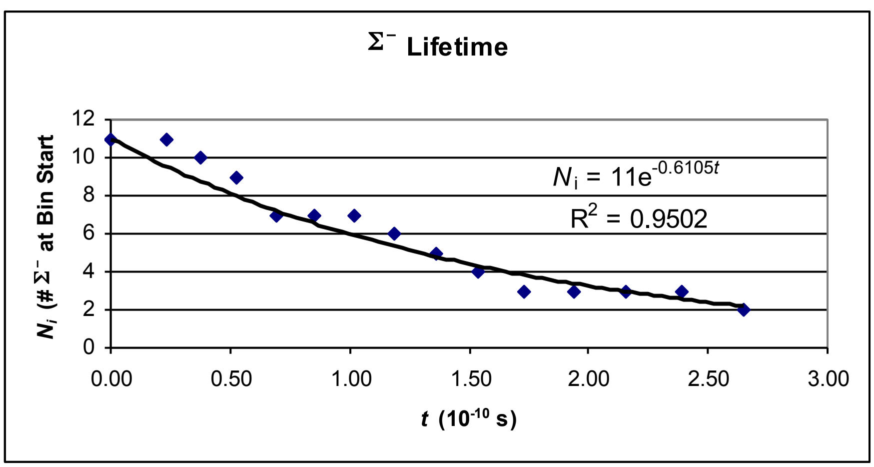
Fit (for N_{i0} and λ): λ = **0.69 x 10^{10} s^{-1}** τ = **1.46 x 10^{-10} s**

Standard: λ **= 0.676 x 10^{10} s^{-1}** τ **= (1.479 ± 0.011) x 10^{-10} s**

S^- Lifetime

Bin Start Time (10^{-10} s)	N_i S^- at Bin Start
0.00	**11**
0.23	**11**
0.38	**10**
0.53	**9**
0.69	**7**
0.85	**7**
1.02	**7**
1.19	**6**
1.36	**5**
1.54	**4**
1.73	**3**
1.94	**3**
2.16	**3**
2.39	**3**
2.65	**2**





ACTIVITY

Student groups need mm rulers and graphing calculators or spreadsheets on computers.

Particle Physics – Σ^- Lifetime

Most of the particles discovered during the 20th century are unstable and spontaneously decay with very short mean lifetimes into less massive particles. Our goal here is to determine the lifetime of one such particle, the Σ^- particle. To produce the Σ^- particles whose tracks are shown in Figures 1 and 2, high-energy protons from the CERN Proton Synchrotron crashed into a stationary target. Among the particles produced were negative kaons (K^-). These kaons were then selected and directed to the Saclay 80-cm bubble chamber. Many of the kaons incident on the chamber had low enough energy that they stopped before interacting with protons in the chamber. In the photos in Figures 1 and 2, a negative kaon (K^-) entered from the bottom and left a trail of ions around which hydrogen bubbles formed. The K^- came to rest and interacted with a proton (also essentially at rest) from the liquid hydrogen. The photos show production of a Σ^- and a π^+, one of several possible results of the kaon-proton interaction.

Explain why you expect the Σ^- and π^+ to have equal and opposite momenta.

Use energy conservation to **show that** Σ^- and π^+ momentum must each be 173 MeV.

Calculate the **energy, kinetic energy,** and **speed** of the Σ^- when it is produced.

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The π^+ and the Σ^- are unstable particles. About 99.99% of all positive pions decay into a positive muon and a muon neutrino ($\pi^+ \rightarrow \mu^+ + \nu_\mu$). The pion mean lifetime is $\tau_\pi = 2.60 \times 10^{-8}$ s. About 99.85% of all Σ^- particles decay into a neutron and a negative pion ($\Sigma^- \rightarrow n^0 + \pi^-$). In this exercise you will measure the Σ^- mean lifetime (τ_{Σ^-}).

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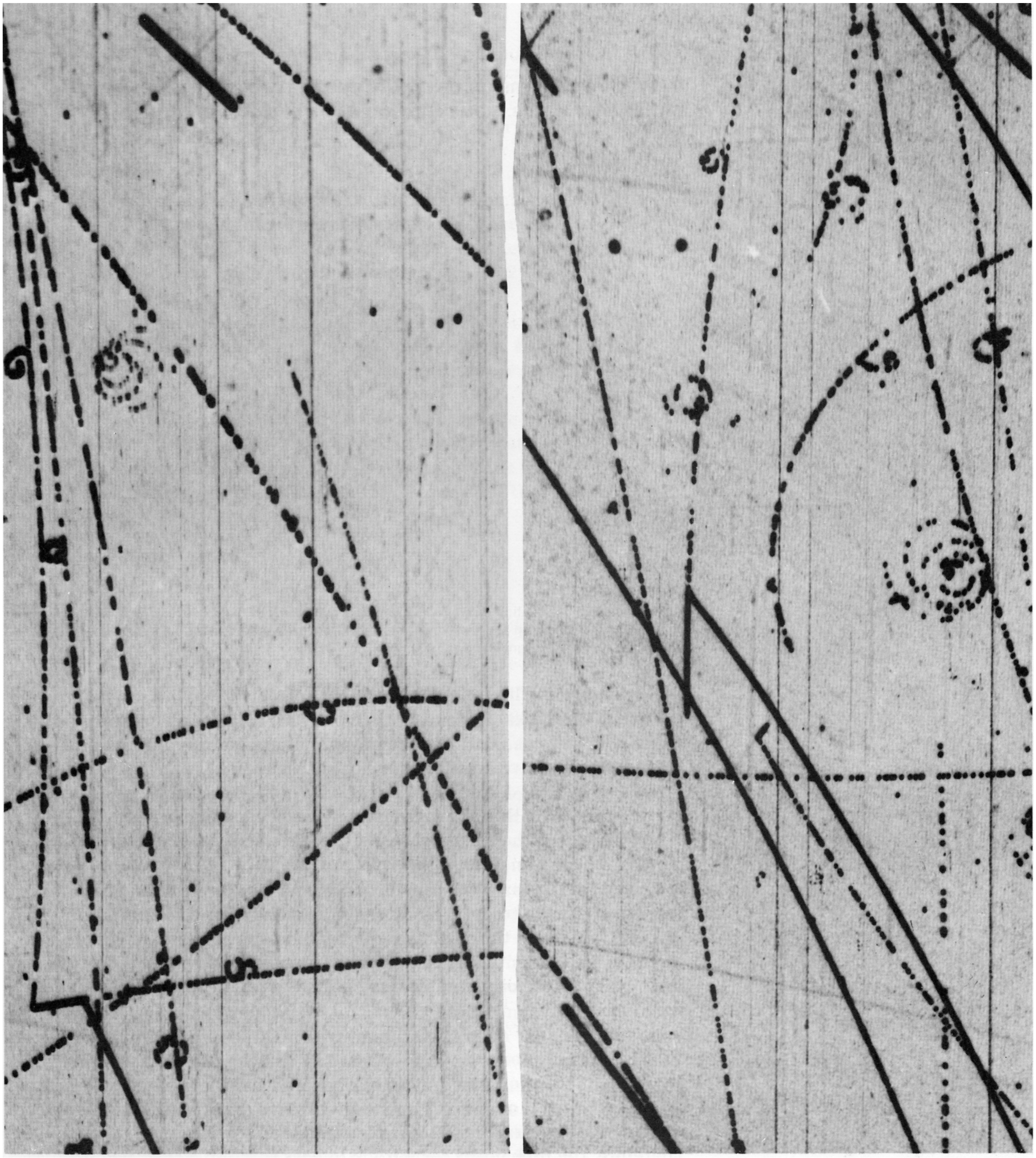
Suppose you have a large collection of N_0 fair 6-sided dice. After each shake of the dice, those showing one spot on the top face are removed.
Write an expression for the number (N) of dice remaining after n shakes.
Calculate the value of the decay constant λ.

We assume a random process of the type indicated above represents particle decay. In this model for unstable particles, a constant fraction of the particles present at any time ($\Delta N/N$) will decay and be removed from the collection during some specified time interval Δt, *i.e.* **($\Delta N/N$)/Δt = λ = decay constant**. The value of λ depends on the nature of the particle. For a system starting with N_0 particles at time t = 0, the number of particles remaining at any later time is given by the expression

$N = N_0 e^{-\lambda t} = N_0 e^{-t/\tau}$, where $1/\lambda = \tau$ = mean lifetime.

By measuring ($\Delta N/N$)/Δt = λ several times for a random collection of one type of particle, one can determine an average value for λ and, consequently, an average value for τ, the mean lifetime of the particle. That is the goal of this exercise.



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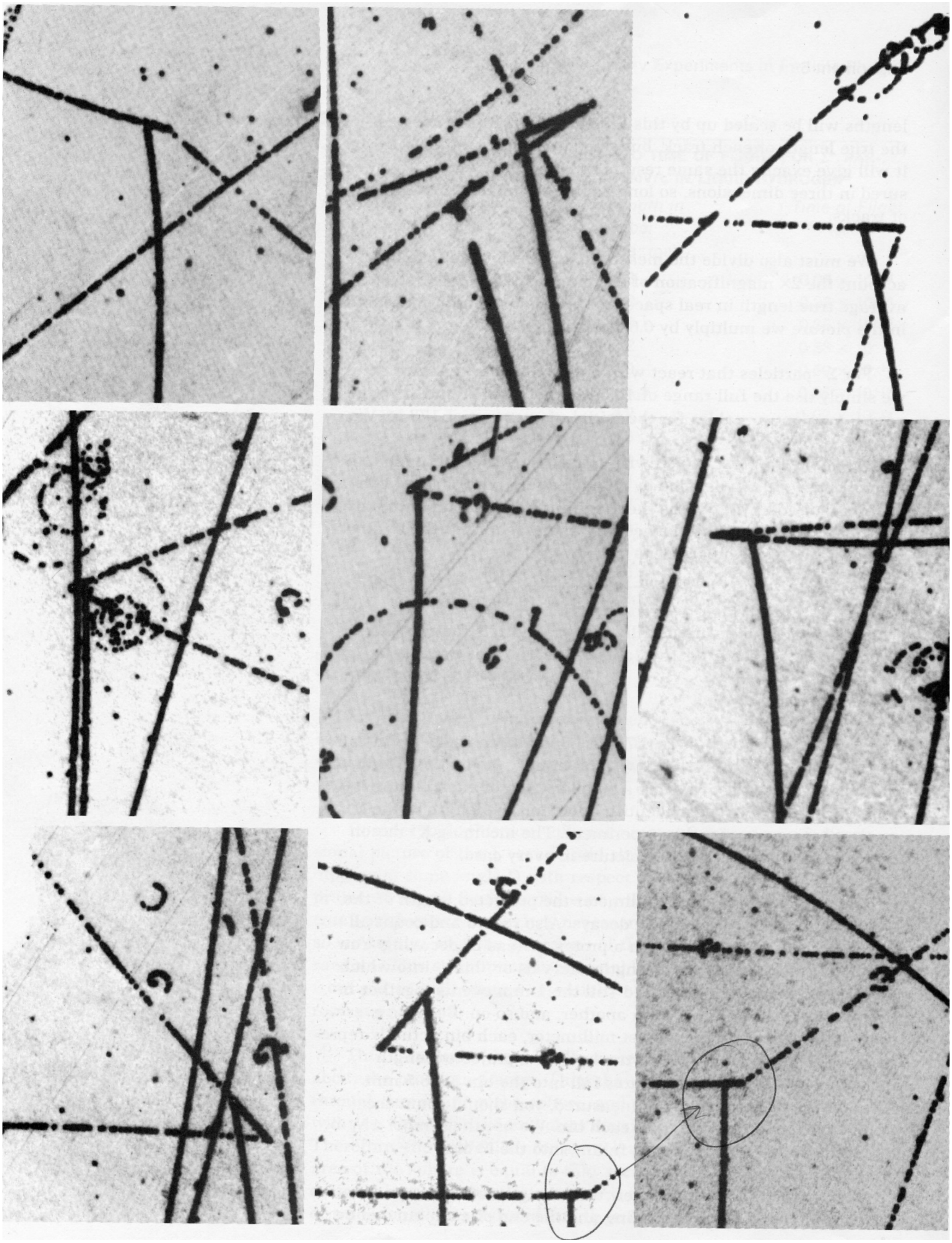

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Since we don't have values for the dip angle for each S^- track, we will adjust each L_M value by dividing it by 2 to compensate for the magnified scale of the photos and dividing by the average value of $\cos\theta$. This should be a reasonable correction to convert L_M values to L_T values. In three dimensions, the average value of $\cos\theta$ is $\pi/4$.

Thus, $L_T = (L_M/2)/(\pi/4) = (2/\pi)L_M = 0.637L_M$.

Table 2. Time of Flight Related to Track Length for S^- Particles with Initial Momentum of 173 MeV

L_M Bin (mm)	L_T Avg. (mm)	Bin Start Time (10^{-10} s)	Bin End Time (10^{-10} s)	Bin Δt (10^{-10} s)	N_i S^- at Bin Start	ΔN_i S^- Decays in Bin
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2.5 – 3.5	1.6 – 2.2	0.38	0.53	0.15		
3.5 – 4.5	2.2 – 2.9	0.53	0.69	0.16		
4.5 – 5.5	2.9 – 3.5	0.69	0.85	0.16		
5.5 – 6.5	3.5 – 4.2	0.85	1.02	0.17		
6.5 – 7.5	4.2 – 4.8	1.02	1.19	0.17		
7.5 – 8.5	4.8 – 5.4	1.19	1.36	0.17		
8.5 – 9.5	5.4 – 6.1	1.36	1.54	0.18		
9.5 – 10.5	6.1 – 6.7	1.54	1.73	0.19		
10.5 – 11.5	6.7 – 7.4	1.73	1.94	0.21		
11.5 – 12.5	7.4 – 8.0	1.94	2.16	0.22		
12.5 – 13.5	8.0 – 8.6	2.16	2.39	0.23		
13.5 – 14.5	8.6 – 9.3	2.39	2.65	0.26		
14.5 – 15.5	9.3 – 9.9	2.65	2.95	0.30		

From your record of measured track lengths (L_M) in Table 1, **enter** the number of decays in each bin (ΔN_i) in Table 2. Then **calculate** and **enter** the N_i values for each bin.

Next, **calculate** from Table 2 and **enter** in Table 3 below the decay constant λ_i for each bin (i) from L_M = 2 mm to L_M = 15 mm $\lambda_i = (\Delta N_i/N_i)/\Delta t_i$.

Track lengths less than 2 mm are omitted from the calculation because such short tracks are difficult to identify with reliability. For tracks longer than 15 mm, the S^- particle is likely to have come to rest and interact with a proton before decaying, so those track lengths are not representative of decay times. Thus, tracks longer than 15 mm are also omitted from the calculation. Some of the λ_i values will be zero.

Calculate the average λ_i value. Include the zero λ_i values in your average.

Average $\lambda = \lambda_{av}$ = ______________________

Calculate the mean lifetime (τ) for the S^- particle and the approximate precision of the measurement $\tau/\sqrt{n}$, where n = number of decays included in the calculation of τ.

Mean lifetime $\tau = 1/\lambda_{av}$ = ___________________ $\pm$ $\tau/\sqrt{n}$ = ____________________

Table 3. Decay Constants for Each Bin

L_M Bin (mm)	$\lambda_i = (\Delta N_i/N_i)/\Delta t_i$ (10^{10} s^{-1})
1.5 – 2.5	
2.5 – 3.5	
3.5 – 4.5	
4.5 – 5.5	
5.5 – 6.5	
6.5 – 7.5	
7.5 – 8.5	
8.5 – 9.5	
9.5 – 10.5	
10.5 – 11.5	
11.5 – 12.5	
12.5 – 13.5	
13.5 – 14.5	
14.5 – 15.5	

Finally, **plot** N_i vs. t for L_M values of 2 mm to 15 mm. **Fit** an exponential curve to the data, and **compare** λ and τ values from the fit to your calculated λ_{av} and τ values and to standard λ and τ values.

Calculated: λ_{av} = ______________ τ = __________________

Fit: λ = ______________ τ = __________________

Standard: λ = 0.676 x 10^{10} s^{-1} τ = (1.479 ± 0.011) x 10^{-10} s

Reflection

Why would the 80-cm bubble chamber used here be inadequate for measuring the lifetime of a particle if the lifetime were 100 times shorter or 100 times longer than the Σ^- lifetime?